CHAPTER 1

Electrostatics

Topic Covered

- Electric Charge
- Electrostatic Force
- Electric Field
- Electric Dipole
- Electric Potential
- Electric Flux
- Gauss's Law
- Capacitor
- Parallel Plate Capacitor
- Energy Stored in Capacitor
- Grouping of Capacitor
- Van De Graaff Generator

1. ELECTRIC CHARGE:

- An Electric Charge is the intrinsic property of the matter. The total deficiency or excess of electrons in a body is the measure of its charge.
- SI unit of electric charge is Coulomb (C).
- Dimensional Formula of Q: [AT]

BASIC PROPERTIES OF ELECTRIC CHARGES:

- Like charges repel and unlike charges attract each other.
- **Additivity: -** According to the additive nature, the total charge on a body is the algebraic sum of all charges located anywhere on the body, provided the sign of the charges is take into account.

  For Example: The resultant of the two charges $+q$ and $+3q$ is $+4q$. On the other hand the resultant of two charges $-q$ and $+4q$ is $+3q$.
- **Relativistic Invariance**: - The numerical value of an elementary charge is **independent of velocity.**
- **Quantization of Charge**: - According to quantization of charge, a body can have only those values of charges that are integral multiples of electronic charge (e).

  i.e., $q = \pm ne$.

  Where n is an integer $\pm 1, \pm 2, \pm 3, ...$ and $e = 1.60 \times 10^{-19}$ C
- **Conservation of charge:** According to the law of conservation of charge, the net charge of an isolated system remains unaltered. Charge can neither be created not be destroyed. It can only be transferred from one part of the system to another, but the net charge remains constant.

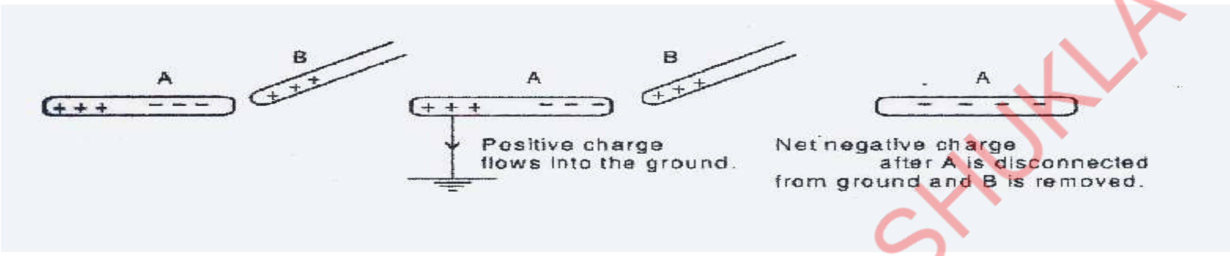
Methods of Charging: - Friction, Induction or conduction.

Friction: - It means that during the process of rubbing, we provide energy which is used to transfer the electrons from one body to another, resulting in charging. For example: When glass is rubbed with silk electrons are transferred from glass to silk. Glass becomes positively charged and silk becomes negatively charged.

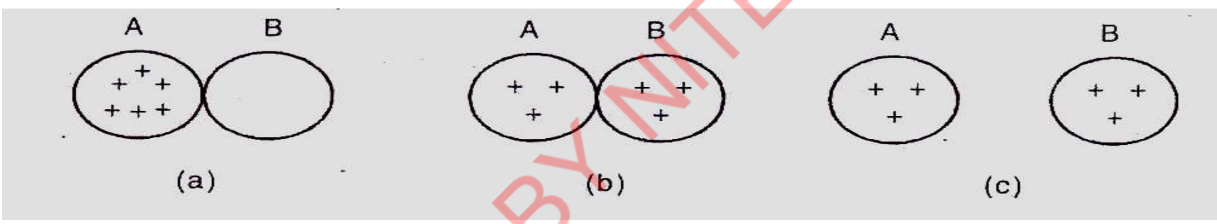
FRICTIONAL CHARGE ACQUIRED IN DIFFERENT PAIRS.

POSITIVE CHARGE	NEGATIVE CHARGE
1. Glass Rod	1. Silk Cloth
2. Fur or woolen cloth	2. Ebonite
3. Dry Hair	3. Comb

Induction:- When a charged body is brought near a neutral one, equal and opposite charges are induced in the neutral body, with opposite charges closer to the charged body and like charges farther away.



Conduction:-Figure below clearly depicts the process of conduction. The basic difference between conduction and induction is that the same type of charge is transferred in conduction while in induction equal and opposite charges are induced in a neutral body.



SOLVED EXAMPLES:

Example.1 A glass rod when rubbed with silk acquires a charge $+1.6\times10^{-19}$ C. What is the charge on the silk?

Solution. If the glass acquires a charge $+1.6\times10^{-19}$ C, then the charge on the silk will be -1.6×10^{-19} C.

Example.2 How many electrons will make a charge of magnitude -8 coulomb?

Solution. Let the number of electrons required to make a charge of magnitude -8 C be n.

According to quantization of charges, $q = ne$.

or number of electrons, $n = \frac{q}{e} = \frac{8}{1.6\times10^{-19}} = 5\times10^{19}$

Example.3 Calculate protonic charge in 100 cc. of water.

Solution. 1 $cc = 1\,gm$ for water as density $= 1000\,kg/m^3$

Now no of atoms in $18\ gm$ (atm weight) $= 6.023\times10^{23}$ and each molecule of H_2O contains 10 proton (8 of oxygen + 2 of H_2)

So No. of proton in $100\ \text{gm}$ water

$$= \left(\frac{6.023\times10^{23}}{10}\times100\right)\times10 = 6.023\times10^{25}$$

Hence protonic charge $= 3.3\times10^{25}\times1.6\times10^{-19} = 5.3\times10^{6}\,C$.

EXERCISE:

1. A body can be negatively charged by
   (1) Giving excess of electrons to it
   (2) Removing some electrons from it
   (3) Giving some protons to it
   (4) Removing some neutrons from it
2. In nature, the electric charge of any system is always equal to
   (1) Half integral multiple of the least amount of charge
   (2) Zero
   (3) Square of the least amount of charge
   (4) Integral multiple of the least amount of charge

2. COULOMB'S LAW:

Two charges exert electric force on each other. The magnitude and direction of force is given by Coulomb's Law. This law states that the force between two charges is proportional to the

(i) product of the magnitude of charges

(ii) reciprocal of the square of the distance between the charges.

The force is directed along the line joining the two charges and is given by

$$F = \frac{kq_1.q_2}{r^2}$$

Here, r is the distance of separation between the two charges of magnitudes q_1 and q_2.

If charges are placed in vacuum $\quad k = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ Nm^2C^{-2}$

PERMITTIVITY:

Permittivity is a measure of the ability of the medium surrounding electric charges to allow electric lines of force to pass through it. It determines the forces between the charges.

ε_0 = permittivity of free space (vacuum)= $8.85\times10^{-12}\ C^2/Nm^2$

Dimensions $[\varepsilon_0] = M^{-1}L^{-3}T^{4}A^{2}$

Relative Permittivity: The relative permittiveity or the dielectric constant (ε_r or K) of a medium is defined as the ratio of the permittivity ε of the medium to the permittivity ε_0 of free space i.e.,

$$\varepsilon_r \text{ or } K = \frac{\varepsilon}{\varepsilon_0}$$

If charges are placed in a medium with permittivity ε, then the forces caused by induced charges in the medium reduce the force between point charges. The new force is,

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

$$F_{medium} = \frac{F_{vacuum}}{\varepsilon_r}$$

The dielectric constants of different mediums are:

Medium	Vacuum	Air	Water	Mica	Teflon	Glass	PVC	Metal
ε_r	1	1.00059	80	6	2	5-10	4.5	∞

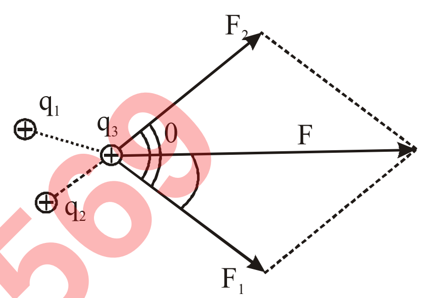
Principle of superposition (Forces):

When many charges are present say $q_1, q_2, q_3 ......$ then net force on any charge, say q_3 will be vector sum of the forces due to other charges q_1, q_2 on q_3

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$$

The resultant of two forcesm $\vec{F_1}$ and $\vec{F_2}$ (see fig.) is determined by using.

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$



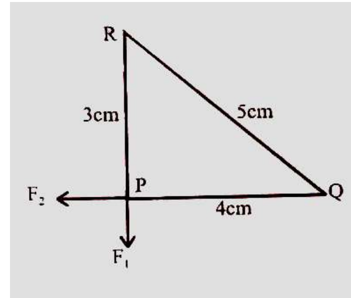
The resultant makes an angle θ with the F_1.

$$\tan\beta = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

SOLVED EXAMPLES:

Example.4 Three equal charges, each having a magnitude of 4.0×10^{-6}C, are placed at the three corners of a right angled triangle of sides 3cm, 4cm and 5 cm. Find the net force on the charge placed at right angle corner.

Solution. In the figure, we wish to find the force on P. The force on P due to R is



$$F_1 = \frac{(4.0\times10^{-6})(4\times10^{-6})}{4\pi\varepsilon_0\times(0.03)^2}$$

$$= \frac{9\times10^{9}\times4^2\times10^{-12}}{9\times10^{-4}} = 160\,\text{N}$$

The force acts along RP

Similarly, the force F_2 on P due to Q is 90N in the direction of RP. Therefore the resultant force is

$$F = \sqrt{F_1^2+F_2^2} = \sqrt{160^2+90^2} = 183.6\,\text{N}$$

The resultant force makes an angle θ with QP given by

$$\theta = \tan^{-1}\left(\frac{90}{160}\right) = \tan^{-1}\left(\frac{9}{16}\right)$$

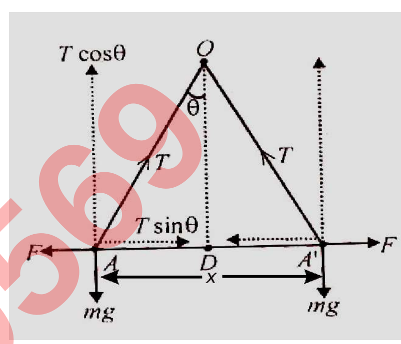
Example.5 Two similar small balls having mass m and charge q are suspended by silk strings having length l, according to the figure. If in the figure θ is very small angle then for equilibrium what will the distance between he centre of the two balls.

Solution. The force acting on the system are as follows.

T is tension in string, F is coulomb force and mg is weight.

For equilibrium,

$$T\ \text{Cos}\theta = mg \ \text{ and } T\ \sin\theta = F$$



$$\therefore\ \tan\theta = \frac{F}{mg} = \frac{1}{4\pi\in_0 x^2}\frac{q^2}{mg}$$

$$\because\ \theta \text{ is very small}$$

$$\therefore\ \tan\theta \approx \sin\theta \approx \frac{x}{2l}$$

$$\therefore\ \frac{x}{2l} = \frac{1}{4\pi\in_0 x^2}\frac{q^2}{mg}$$

$$x = \left\{\frac{q^2 l}{2\pi\in mg}\right\}^{1/3}$$

Example.6 In a medium, the force between two point charges placed at a distance d apart is F. What should be their separation in the same medium if the force between them is to be

(a) $2F$ (b) $F/3$

Solution. $F \propto \frac{1}{d^2}$

Therefore,

(a) $d_1 = \frac{d}{\sqrt{2}}$ (b) $d_2 = \sqrt{3}\,d$

EXERCISE:

3. There are two charges $+1\mu C$ and $+5\mu C$ respectively. The ratio of the forces acting on them will be

(1) 1 : 5 (2) 1 : 1

(3) 5 : 1 (4) 1 : 25

4. A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when

(1) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$ (2) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$

(3) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{3}$ (4) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

5. The charges on two spheres are +7 μC and –5 μC respectively. They experience a force F. If each of them is given and additional charge of –2 μC, the new force of attraction will be

(1) F (2) F/2

(3) $F/\sqrt{3}$ (4) 2F

3. ELECTRIC FIELD:

The region or space around a charged body within which its influence can be felt is called electric field.

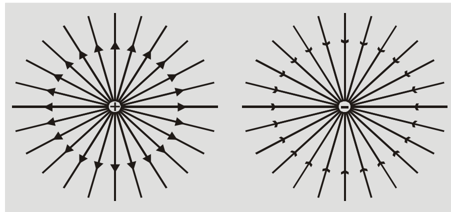
The electric field $\vec{E}$ at any point is equal to the force experienced per unit (test) charge placed at that point, and is directed along the direction of the force experienced

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric field (intensity) is a vector quantity its direction is same as the Force **Unit of [E]:** newton/coulomb or volt/metre

Dimensions of [E]: $\frac{[F]}{[q]} = \frac{MLT^{-2}}{TA} = MLT^{-3}A^{-1}$

ELECTRIC LINES OF FORCE:

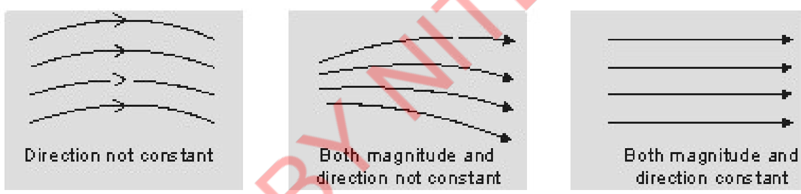


[1] Electric lines of force usually start or diverge out form positive charge and end or converge on negative charge.

[2] Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

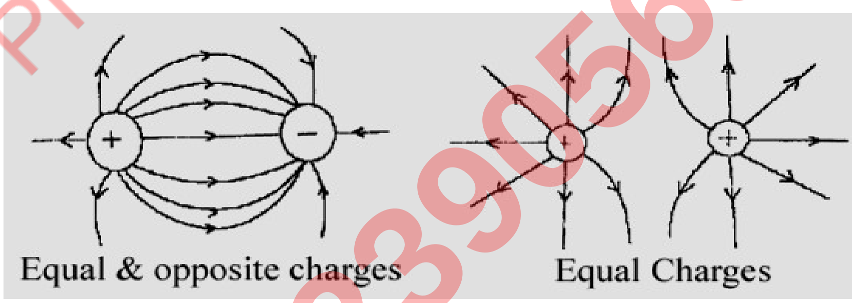
[3] Lines of force never cross each other because if they will cross then intensity at that point will have two directions which is not possible.

[4] The relative density or closeness of the field lines at different points indicates the relative strength of electric field at those points. The field lines crowded where the field is strong and are spaced apart where it is weak.



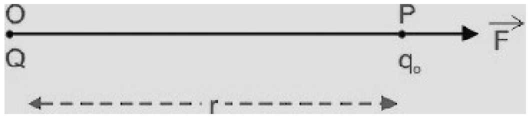
[5] The number of lines leaving a positive point charge or entering a negative charge is proportional to the charge. (if a charge q gets 8 lines, charge 2q deserves 16 lines).

[6] The lines of force of like chages repel each other and unike charges attract each other.



[7] Electric lines of force end or start normally on the surface of a conductor.

ELECTRIC FIELD DUE TO A POINT CHARGE:



Consider a single point charge q at O. If we place a test charge q_0 at some point P given by the position vector $\vec{r}$, then force on the charge is

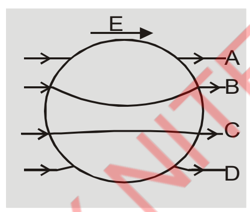
$$F = \frac{q\, q_0}{4\pi\varepsilon_0 r^2}\hat{r} \qquad As \quad \vec{E} = \frac{\vec{F}}{q_0}$$

$$\therefore \qquad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

SOLVED EXAMPLES:

Example.7 A Solid metallic sphere is placed in a uniform electric field. Which of the lines A, B, C and D shows the correct path and why ?

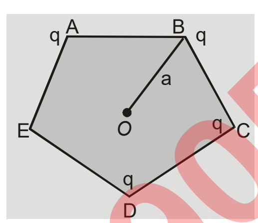
Solution. Path (A) is wrong as lines of force do start or end normally on the surface of a conductor. Path (B) and (C) are wrong as lines of force should not exist inside a conductor.



Also lines of force are not normal to the surface of conductor. Path (D) represents the correct situation as here line of force does not exist inside the conductor and starts and ends normally on its surface.

Example.8 Charges with magnitude q are placed at 4 corners of a regular pentagon. These charges are at distance ‘a’ from the centre of the pentagon. Find electric field intensity at the centre of the pentagon.

Solution. Charges are placed at corners A, B, C and D of the pentagon. If charge q is placed at the fifth corner also then by symmetry the intensity $\vec{E}$ at centre O is zero.



Electric field at O due to new charge placed at $E = \dfrac{q}{4\pi \in_0 a^2}$ along $\overrightarrow{EO}$

$\therefore$ Electric field due to charges at A, B, C and D.

$$E = \frac{q}{4\pi \in_0 a^2} \text{ along } \overrightarrow{OE}.$$

Example.9 Along $x-axis$ at positions $x=1,\ x=2, x=4$ and $x=8$ charges q is placed. What will be electric field at $x=0$ due to these charges. What will be the value of electric field if the charges are alternately positive and negative.

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x = 1 x = 2 x = 4 x = 8

Solution. By superposition theory

$$E=\frac{q}{4\pi\in_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+.....\right]$$

$$=\frac{q}{4\pi\in_0}\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....\right]$$

Terms in the braket are G.P. with first term $a=1$ and common ratio $r=\frac{1}{4}$. Its sum $S=\frac{a}{1-r}$

$$\therefore\ E=\frac{q}{4\pi\in_0}\left[\frac{1}{1-1/4}\right]=\left[\frac{1}{4\pi\in_0}\right]\frac{4}{3}q$$

If the charges are alternately positive and negative

$$E=\frac{q}{4\pi\in_0}\left[1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+....\right]$$

Where $a=1, r=-1/4$

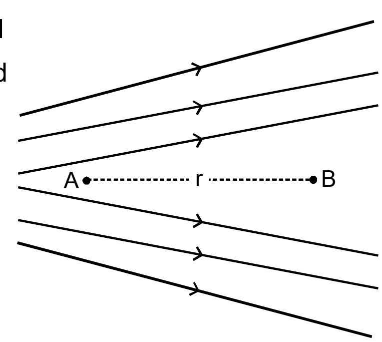
$$E=\frac{q}{4\pi\in_0}\left[\frac{1}{1-(-1/4)}\right]=\left[\frac{q}{4\pi\in_0}\right]\frac{4q}{5}$$

EXERCISE:

6. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium, if q is equal to

(1) $-\frac{Q}{2}$ (2) $-\frac{Q}{4}$ (3) $+\frac{Q}{4}$ (4) $+\frac{Q}{2}$

7. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the displacement between A and B is r then



(1) $E_A > E_B$ (2) $E_A < E_B$

(3) $E_A = \frac{E_B}{r}$ (4) $E_A = \frac{E_B}{r^2}$

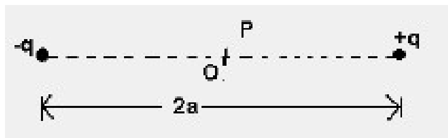
8. The magnitude of electric field intensity E is such that, an electron place in it would experience an electrical force equal to its weight is given by

(1) mge (2) $\frac{mg}{e}$ (3) $\frac{e}{mg}$ (4) $\frac{e^2}{m^2}g$

4. ELECTRIC DIPOLE:

An arrangement of two equal and opposite charges separated by a fixed distance is known as electric dipole.

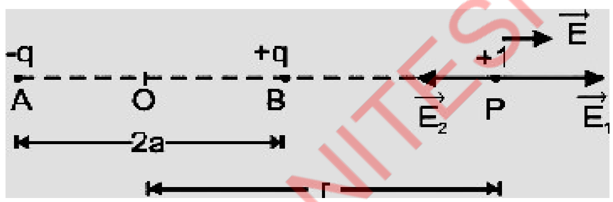
Dipole moment of the Dipole is define as:



$p =$ charge ×distance between the charges = $q \times 2a$

Dipole moment is a vector quantity and is directed from $-ve$ charge towards the $+ve$ charge. The line through q and $-q$ is known as the ***axis of the dipole.*** Its unit is ***Coulomb meter.***

ELECTRIC FIELD DUE TO A DIPOLE ON THE AXIAL LINE, END ON POSITION:



Considering an electric dipole of two point charges –q and +q, separated by distance 2a apart.We have to calculate the electric field intensity at point P on the axial line, at a distance r from the centre. E_1 and E_2 are electric field at P due to charge $-q$ and $+q$ respectively.

Resultant intensity $\vec{E}$ at P is

$$|\vec{E}| = |\vec{E_1}| - |\vec{E_2}|$$

$$= \frac{q}{4\pi\varepsilon_0 (r-a)^2} - \frac{q}{4\pi\varepsilon_0 (r+a)^2}$$

$$= \frac{q \times 2a \times 2r}{4\pi\varepsilon_0 (r^2 - a^2)^2}$$

$$|\vec{E}| = \frac{|\vec{p}|\, 2r}{4\pi\varepsilon_0 (r^2 - a^2)^2}$$

For $2a << r$

$E = \frac{1}{4\pi_0} \frac{2p}{r^3}$, along the Dipole moment

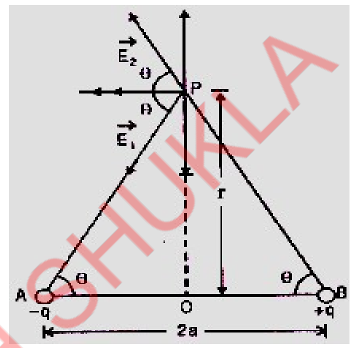
FIELD DUE TO AN ELECTRIC DIPOLE ON ITS EQUATORIAL LINE, BROAD SIDE ON POSITION:

Consider a dipole AB Consisting of the charges -q and +q separated by a distance 2a. Point P is located on the equatorial line at a distance r from the mid-point O of the dipole AB.

From the figure.

If $\vec{E}_1$ is the electric field intensity at P due to charge -q at A. then

$$|\vec{E}_1| = \frac{q}{4\pi\varepsilon_0 AP^2} = \frac{q}{4\pi\varepsilon_0 (r^2+a^2)}$$



$\vec{E}_2$ is the electric fieldintensity at P due to charge +q at B, then

$$|\vec{E}_2| = \frac{q}{4\pi\varepsilon_0 BP^2} = \frac{q}{4\pi\varepsilon_0 (r^2+a^2)}$$

Thus, resultant electric field at point P is,

$$|\vec{E}| = E_1 \cos\theta + E_2 \cos\theta$$

$$|\vec{E}| = 2E_1 \cos\theta \left(|\vec{E}_1| = |\vec{E}_2|\right)$$

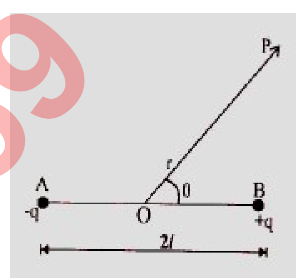
$$|\vec{E}| = \frac{2q}{4\pi\varepsilon_0 (r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}, \quad p = q \times 2a$$

For $2a << r$,

$$E = \frac{p}{4\pi\varepsilon_0 r^3}$$ is opposite to Dipole moment.

ELECTRIC FIELD AT ANY POINT $P(r, \theta)$ DUE TO SHORT DIPOLE:



Let P be a point at a distance r from the mid-point O of the short dipole. Let θ be the angle between OP and the dipole moment p.

Then electric field at P can be shown to be given by

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

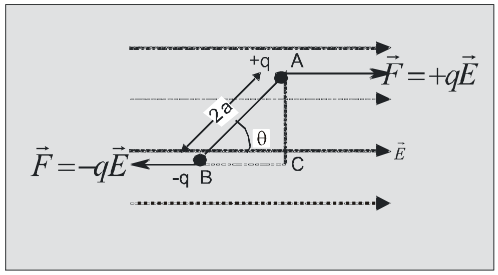
TORQUE EXPERIENCED BY ELECTRIC DIPOLE IN UNIFORM ELECTRIC FIELD:

The magnitude of torque is given by;

Torque, τ = Force x arm of the couple = F × AC = (qE) 2a $\sin\theta$

τ = pE $\sin\theta$ (p = q × 2a)

$\vec{\tau} = \vec{p} \times \vec{E}$



- When $\theta = 0^{o}$, the dipole moment $\vec{p}$ is in the direction of the field $\vec{E}$ and the dipole is in stable equilibrium. If it is slightly displaced, it performs oscillations.
- When $\theta = 180^{o}$, the dipole moment $\vec{p}$ is opposite to the direction of the field $\vec{E}$ and the dipole is in unstable equilibrium.
- When a dipole is kept in a non-uniform field, the charges of the dipole experience unequal force, therefore, the net force on the dipole in not equal to zero. The magnitude of the force isgiven by negative derivative of the potential energy with respect to distance along the axis of the dipole

$$F = -\frac{dU}{dx} = -p.\frac{dE}{dx}$$

POTENTIAL ENERGY OF DIPOLE:

Let a dipole with dipole moment $\vec{p}$ makes an angle θ with the direction of uniform external electric field $\vec{E}$ from position of stable equilibrium.

The work done in rotating the dipole through an infinitesimal angle $d\theta$ against a torque is given by,

$dW = \tau d\theta = pE \sin\theta d\theta$

Now the total work done in rotating the dipole from the orientation θ_1, θ_2 can be written as,

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta = pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

or $W = -pE[\cos\theta_2 - \cos\theta_1]$ .....(i)

Let the dipole be initially at right angle to $\vec{E}$ i.e. $\theta = 90^{o}$ and it is to be set at $\angle\theta$ with $\vec{E}$ i.e. $\theta_2 = \theta$

Substituting in equation (i)

$W = -pE[\cos\theta - \cos 90^{o}] = -pE\cos\theta$ .... (ii)

This work done is stored in the dipole. The form of energy is termed as potential energy (U) of the dipole.

U = $-pE\cos\theta$

1. $\theta = 0$, $U_{min} = -pE$, Stable Equilibrium.
2. $\theta = \pi$, $U_{max} = pE$, Unstable Equilibrium.

PROBLEMS RELATED TO DIPOLE:

SOLVED EXAMPLES:

Example.10 Calculate the electric intensity due to a dipole of length 10 cm and having a charge of 500 mC at a point on the axis, 20 cm from one of the charges in air .

Solution. The electric intensity on the axial line of the dipole

$$E = \frac{1}{4\pi \in_0} \frac{2pd}{\left(d^2 - l^2\right)^2}$$

$2l$ = 10 cm $\therefore l$ = 5 × 10^{-2} m

d = 20 + 5 = 25 cm = 25 × 10^{-2} m

p = 2ql = 2 × 500 × 10^{-6} × 5 × 10^{-2} $\Rightarrow$ 5 × 10^{-3} × 10^{-2} = 5 × 10^{-5} cm

$$E = \frac{9\times10^9 \times 2\times5\times10^{-5}\times25\times10^{-2}}{10^{-8}\left[25^2 - 5^2\right]^2}$$

$$\Rightarrow 6.25\times10^7 \text{ N.C.}$$

Example.11 Calculate the electric intensity due to an electric dipole of length 10 cm having charges of 100 μC at a point 20 cm from each charge.

Solution. The electric intensity on the equatorial line of an electric dipole is

$$E = \frac{1}{4\pi \in_0} \frac{p}{\left(d^2 + l^2\right)^{3/2}}$$

$$p = 2l\ q \text{ C-m}$$

$$= 10\times10^{-2}\times100\times10^{-6}$$

$$= 10^{-5} \text{ C-m}$$

$$d^2 + l^2 = \left(20\times10^{-2}\right)^2 = 4\times10^{-2}$$

$$\therefore E = \frac{9\times10^9\times10^{-5}}{\left(4\times10^{-2}\right)^{3/2}}$$

$$= \frac{9\times10^9\times10^{-5}}{10^{-3}\times8} = \frac{9}{8}\times10^7 = 1.125\times10^7 \text{ N/C}$$

Example.12 Find out torque on dipole in N-m given: Electric dipole moment $\vec{P} = 10^{-7}\left(5\hat{i} + \hat{j} - 2\hat{k}\right)$ coulomb metre and electric field $\vec{E} = 10^7\left(\hat{i} + \hat{j} + \hat{k}\right)$ Vm^{-1} is -

Solution. $$\hat{\tau} = \vec{P}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1+2) + \hat{j}(-2-5) + \hat{k}(5-1) = 3\hat{i} - 7\hat{j} + 4\hat{k},$$

$$\left|\hat{\tau}\right| = 8.6 \text{ N-m}$$